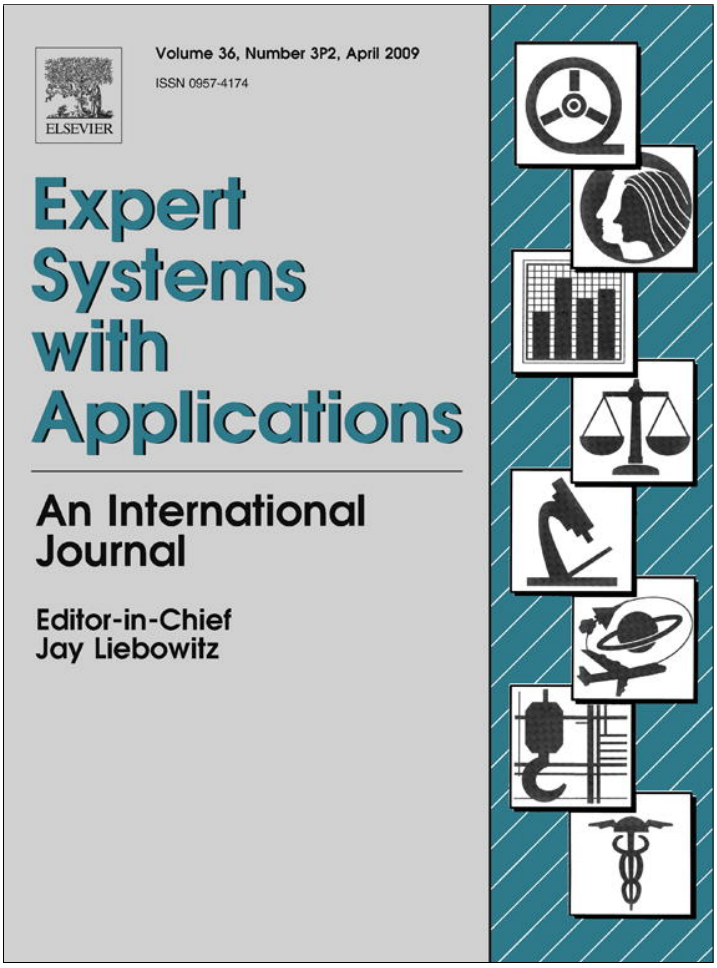

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LMI static output-feedback design of fuzzy power system stabilizers

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A R T I C L E   I N F O

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A B S T R A C T

The design of a model-free fuzzy power system stabilizer (PSS) lacks systematic stability analysis and performance guarantees. This paper provides a step towards the design of a model-based fuzzy PSS that guarantees not only stability but also performance specifications of power systems. A new practical and simple design based on static output feedback is proposed. The design guarantees robust pole-clustering in an acceptable region in the complex plane for a wide range of operating conditions. A power system design model is approximated by a set of Takagi–Sugeno (T–S) fuzzy models to account for nonlinearities, uncertainties and large scale power systems. The proposed PSS design is based on parallel distributed compensation (PDC). Sufficient design conditions are derived as linear matrix inequalities (LMI). The design procedure leads to a tractable convex optimization problem in terms of the stabilizer gain matrix. Simulations results of both single-machine and multi-machine power systems confirm the effectiveness of the proposed PSS design.

1. Introduction

POWER system stabilizers (PSSs) have been used by utilities to damp out the electro-mechanical oscillations that follow disturbances (DeMello & Concordia, 1969; Kundur, 1994). Disturbances occur in power systems due to several reasons, e.g. continuous load variations, set-point changes and faults. In such cases, a fixed-parameter conventional PSS may fail to maintain stability or lead to a degraded performance (Klein, Rogers, Moorty, & Kundur, 1992; Larsen & Swann, 1981). Different design techniques such as adaptive control (Ghosh, Ledwich, Malik, & Hope, 1984; Sastry & Bodson, 1989) and robust control (Klein, Le, Rogers, Farrokhpay, & Balu, 1995; Samarasighe & Pahalawaththa, 1997) have been proposed to enhance the performance of PSSs. The implementation of an adaptive controller needs tough precautions to assure persistent excitation conditions and performance during the learning phase (Sastry & Bodson, 1989).

Recently, fuzzy logic has emerged as a potential technique for PSS design. Besides its ability to accommodate the heuristic knowledge of a human expert, the advantage of a fuzzy PSS is that it represents a nonlinear mapping that can cope with the nonlinear nature of power systems. Several reported results confirm that a fuzzy PSS outperforms a conventional PSS once the deviation from the nominal design conditions becomes significant (Malik & El-Metwally, 1998). Implementation of a fuzzy PSS for a multi-machine power system is reported in El-Metwally and Malik (1996). Tuning the scaling factors of a fuzzy PSS is discussed in El-Metwally and Malik (1993). An adaptive PSS using on-line self-learning fuzzy systems is discussed in Elshafei, El-Metwally, and Shaltout (2005) and on-line tuning of fuzzy PSSs as a direct adaptive one is reported in Abdelazim and Malik (2003). Although the performance of a well-designed model-free fuzzy PSS is acceptable, it lacks systematic stability analysis and controller synthesis. The reported work attempts to overcome this drawback by a providing a model-based fuzzy PSS that guarantees stability and performance of power systems. In the past ten years, research efforts on fuzzy logic control have been devoted to model-based fuzzy control systems (Feng, 2006). Stability and performance limits of model-based fuzzy control systems can be achieved via linear matrix inequality (LMI) techniques (Tanaka & Wang, 2001).

LMI techniques are proposed as design tools of robust PSS in Befekadu and Erlich (2006); Rao and Sen (2000); Ramos, Alberto, and Bretas (2003); Werner, Korba, and Chen Yang (2003). In Werner et al. (2003), the authors represent the model uncertainty as a linear fractional transformation. An output feedback PSS is designed to guarantee stability for all admissible plants such that a quadratic performance index, based on the nominal plant, is minimized. In Rao and Sen (2000), pole clustering is used to design a full state feedback for a multi-machine power system. In Ramos et al. (2003), a combination of LMI and feedback linearization techniques is used to design a centralized PSS for a two-area power system. In Befekadu and Erlich (2006), a robust decentralized PSS is derived by minimizing a linear objective function under LMI and bilinear matrix inequality (BMI) constraints.

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Nomenclature

V_t	terminal voltage
E_q	induced EMF proportional to field current
E_{fd}	generator field voltage
V_{ref}	the reference voltage
x_e	equivalent tie-line reactance
x'_d,x_d,x_q	generator direct-axis transient reactance, direct and quadrature synchronous reactances
δ	angle between q-axis and infinite bus bar
I_d,I_q	direct and quadrature stator currents
$\Delta\omega$	speed deviation
ω_o	synchronous speed (rad/s)
T_e,T_m	electrical torque and mechanical torque
T'_{d0}	open-circuit d-axis transient time constant
M	inertia coefficient in seconds
K_E,T_E	exciter gain and time constant
V^{∞}	infinite bus bar voltage
P,Q	active and reactive power loading, respectively
s, C	Complex operator and complex plane respectively
$\otimes$	Kronecker product
$X > 0$,$X \geqslant 0$	positive definite and positive semi-definite respectively

In this work, an LMI design of a model-based fuzzy static PSS is proposed. The design guarantees robust pole-clustering in a pre-specified LMI region. The LMI region is selected such that common specifications of power system stabilization are achieved. This includes adequate damping and acceptable speed of the time response over wide ranges of active power (P), reactive power (Q) and tie-line reactance (X_e). These ranges are selected to include all practical loading conditions and very weak to very strong transmission networks. A power system design model is approximated by a polytopic Takagi–Sugeno (T–S) fuzzy model. Each fuzzy rule (vertex) of the T–S model (polytope) represents an extreme operating point corresponding to the selected ranges. According to the universal approximation theorem (Tanaka & Wang, 2001), the resulting fuzzy model can approximate the original nonlinear system to an arbitrary degree of accuracy. A stabilizer design is carried out at each vertex of the polytope. The designs are derived under global stability and performance conditions using a common Lyapunov function. The design leads to a set of LMIs. The solution of this set of LMIs yields a common positive definite matrix that is used to calculate the stabilizer gains. The total control signal is calculated using a PDC control law (Wang, Tanaka, & Griffin, 1995).

Up to our knowledge, application of a model-based fuzzy control in PSS design, as proposed here, is a novel approach. Model-based fuzzy control system allows us to use an imprecise design model (Sugeno, 1999; Sugeno & Kang, 1986; Takagi & Sugeno, 1985). It also enables a decentralized design approach that is independent of the power system size as indicated in the next sections. Furthermore, model-based design relies on LMIs rather than bilinear matrix inequalities (BMIs) to have a tractable solution.

This paper lies in seven sections. Section 2 describes how to represent an uncertain power system that allows a wide range of operating conditions. In Section 3, a brief review of T–S models is depicted followed by the model to be used for PSS design. In Section 4, the LMI conditions that correspond to robust pole clustering are recalled. Consequently, the sufficient conditions required to calculate the fuzzy observer and stabilizer gains are derived in Section 5. In Section 6, simulation results illustrate the merits of our proposed design. A single-machine model is used first to clarify the design steps. Then, a bench mark model of a 4-machine 2-area test system is utilized to compare the proposed PSS to a well-designed conventional PSS. Section VII concludes this work.

2. Deriving the T–S fuzzy model for the proposed PSS design

2.1. A review of T–S fuzzy models and PDC

A T–S fuzzy model (Takagi & Sugeno, 1985), also called type-III fuzzy model by Sugeno (1999), is in fact a fuzzy dynamic model (Cao, Rees, & Feng, 1995, 1997, 1997). This model is based on using a set of fuzzy rules to describe a global nonlinear system by a set of local linear models which are smoothly connected by fuzzy membership functions. T–S fuzzy models include two kinds of knowledge: one is qualitative knowledge represented by fuzzy IF-Then rules, and the other is a quantitative knowledge represented by local linear models. Identification of T–S fuzzy models has been extensively addressed in literature, e.g. Takagi and Sugeno (1985), Sugeno and Kang (1986), Cao, Rees, and Feng (1997) Johansen, Shorten, and Murray-Smith (2000). There are basically two classes of algorithms to identify T–S fuzzy models. The first is to linearize the original nonlinear system in a number of operating points when the model is known. This is adopted in this study. The second is based on the data gathered from the nonlinear system when the model is unknown. The i^{th} rule of a T–S fuzzy model is written as follows:

Model Rule i:

$$\begin{array}{ll} \text{IF} & z_1(t)\ is\ M_1^i\ AND\ldots AND\ z_n(t)\ is\ M_n^i \\ \text{THEN} & \dot{x}(t) = A_i x(t) + B_i u(t) \\ & y(t) = C_i x(t) \end{array}$$

$M_j^i, j = 1, 2, \ldots, n$, is the j^{th} fuzzy set of the i^{th} rule and $z_1(t), \ldots, z_n(t)$ are known premise variables that may be functions of state variables, external disturbances, and/or time. Let $\mu_j^i(z_j)$ be the membership function of the fuzzy set M_j^i and let

$$h_i = h_i(t) = \Pi_{j=1}^n \mu_j^i(z_j)$$

Given a pair ($z(t)$,$u(t)$), the resulting fuzzy system is inferred as the weighted average of the local models and has the following form

$$\begin{aligned} \dot{x} &= \sum_{i=1}^r h_i\{A_i x(t) + B_i u(t)\} / \sum_{i=1}^r h_i = \sum_{i=1}^r \alpha_i\{A_i x(t) + B_i u(t)\} \\ y &= \sum_{i=1}^r \alpha_i C_i x(t) \end{aligned} \tag{1}$$

where, $\alpha_i = h_i / \sum_{i=1}^r h_i, 0 \leqslant \alpha_i \leqslant 1, \sum_{i=1}^r \alpha_i = 1$, for $i = 1,2,\ldots,r$.

The PDC offers a procedure to design a fuzzy controller from a given T–S fuzzy model (Wang et al., 1995; Tanaka & Sugeno, 1992). In the PDC design, each control rule is associated with the corresponding rule of a T–S fuzzy model. The designed fuzzy controller shares the same fuzzy sets with the fuzzy model in the premise parts. For a T–S fuzzy model as described in (1), the following state feedback fuzzy controller is constructed via PDC as follows:

Model Rule # i:

$$\begin{array}{ll} \text{IF} & z_1(t)\ is\ M_1^i\ AND\ldots AND\ z_n(t)\ is\ M_n^i \\ \text{THEN} & u(t) = F_i x(t), i = 1, 2, \ldots, r \end{array}$$

The fuzzy control rules have a linear controller in the consequent parts and the overall fuzzy controller is represented by

$$u(t)=\sum_{i=1}^{r}h_iF_ix(t)/\sum_{i=1}^{r}h_i=\sum_{i=1}^{r}\alpha_iF_ix(t) \tag{2}$$

Although the fuzzy controller (2) is constructed using local design structures, the feedback gains must be determined using global design conditions to guarantee global stability and performance. The methods for stability analysis and control design of T–S fuzzy systems are classified into different categories as reported in Feng (2006). The analysis adopted in this paper requires that a common quadratic Lyapunov function can be found for all the local subsystems in a T–S fuzzy model (Akar & Ozguner, 2000; Akhenak, Chadli, & Ragot, 2004; Bergsten, Palm, & Driankov, 2002; Chadli, Maquin, & and Ragot, 2004; Kang, Lee, & Pusan, 1998; Tanaka & Sugeno, 1992; Tanaka & Wang, 1997; Tanaka, Ikeda, & Wang, 1998).

Substituting (2) in (1), the augmented system is given by

$$\dot{x}=\sum_{i=1}^{r}\sum_{j=1}^{r}\alpha_i\alpha_j\{A_i+B_iF_j\}x(t)$$

Denoting $G_{ij}=A_i+B_iF_j$,

$$\dot{x}=\sum_{i=1}^{r}\alpha_i^2G_{ii}x(t)+2\sum_{i=1}^{r}\sum_{i<j}^{r}\alpha_i\alpha_j\left(\frac{G_{ij}+G_{ji}}{2}\right)x(t) \tag{3}$$

Theorem 1. *the T–S fuzzy model (3) is globally asymptotically stable if there exists a common positive definite matrix X such that*

$$G_{ii}^{T}X+XG_{ii}<0,\quad i=1,2,\ldots,r \tag{4}$$

$$\left(\frac{G_{ij}+G_{ji}}{2}\right)^{T}X+X\left(\frac{G_{ij}+G_{ji}}{2}\right)\leqslant 0,\quad i<j,\quad \alpha_i\cap\alpha_j\neq\varphi \tag{5}$$

Proof. see Tanaka and Wang (2001). □

Corollary 1. *Assume that $B_i=B, i=1,2,\ldots,r$, the equilibrium of the fuzzy control system (3) is globally quadratically stable if a common positive definite matrix P exists and satisfies (4) only. This follows directly because definite negativity of (4) implies semi-definite negativity of (5) in case of common B (Tanaka & Wang, 2001).*

2.2. Power system uncertainties

Power systems consist mainly of a set of generating units, a transmission network and loads. These units interact with each other through active and reactive power generation (P,Q) over the transmission network. Briefly, any power system is composed of a set of inherently interacting subsystems, where each subsystem consists of a generating-unit connected to the rest of the system by a tie line whose reactance is the Thevenin's reactance at the terminal bus $(X_e=X_{Th})$. For modeling and design approaches proposed in this work, a subsystem is considerably approximated by a single-machine connected to an infinite system. This assumption is made possible because fuzzy modeling allows imprecision (Sugeno, 1999; Sugeno & Kang, 1986; Takagi & Sugeno, 1985). As a result of this approximation, each generator can be decoupled from the entire system. The influence of the rest of the system will be taken care of by the scheduling variables; namely its real and reactive powers (P,Q) and an equivalent tie line reactance (X_e). All possible dynamics at the interface between a generator and the rest of the system are supposed to be reflected by this set of scheduling variables (P,Q,X_e). This decoupling leads to a decentralized design.

The origin of power systems uncertainties are the continuous variations in load patterns and transmission network. Since the system is to be linearized around the equilibrium point, it follows that a different system triple (A,B,C) is obtained for each operating point. It is assumed that the set of variables (P,Q,X_e) of certain subsystem varies independently over the following ranges: $P\in\left[\overset{-}{P}\ \ \overset{+}{P}\right]$, $Q\in\left[\overset{-}{Q}\ \ \overset{+}{Q}\right]$ $X_e\in\left[\overset{-}{X}_e\ \ \overset{+}{X}_e\right]$. These ranges are selected to encompass all practical operating points and very weak to very strong transmission networks. Possible combinations of minimum and maximum values of these variables result in eight operating points corresponding to the vertices of a cuboid in the (P,Q,X_e) space. Consequently, a set of matrices obtained from an operating point can be represented by $(A,B,C)\in\Omega$, where:

$$\Omega=\left\{(A,B,C):(A,B,C)=\sum_{i=1}^{8}\alpha_i(A_i,B_i,C_i),\alpha_i\geqslant 0,\sum_{i=1}^{8}\alpha_i=1\right\} \tag{6}$$

The set Ω describes a polytope with eight vertices (A_i,B_i,C_i), $i=1,2,\ldots,8$ calculated at $[\overset{-}{P},\overset{-}{Q},\overset{-}{X}_e]$, $\left[\overset{-}{P},\overset{-}{Q},\overset{+}{X}_e\right]$, …, $\left[\overset{+}{P},\overset{+}{Q},\overset{+}{X}_e\right]$ respectively. Changes in load and system topology or most of system parameters lead to uncertainties in the state-matrix A. Uncertainties in the input matrix B can only be caused by parametric variations in the excitation system and are not taken into account in this work. Rotor speed deviation is selected as the measured output and then no uncertainties appear in matrix C.

2.3. Dynamic T–S Fuzzy Model

Each vertex system in the polytope (6) corresponds to a model rule in a T–S fuzzy system which is stated as follows.

Model Rule 1:

IF $(P\text{ is about }\overset{-}{P})AND(Q\text{ is about }\overset{-}{Q})AND(X_e\text{ is about }\overset{-}{X}_e)$

THEN $\begin{bmatrix}\dot{x}\\ y\end{bmatrix}=\begin{bmatrix}A_1 & B\\ C & 0\end{bmatrix}\begin{bmatrix}x\\ u\end{bmatrix}$

Model Rule 2:

IF $(P\text{ is about }\overset{-}{P})AND(Q\text{ is about }\overset{-}{Q})AND(X_e\text{ is about }\overset{+}{X}_e)$

THEN $\begin{bmatrix}\dot{x}\\ y\end{bmatrix}=\begin{bmatrix}A_8 & B\\ C & 0\end{bmatrix}\begin{bmatrix}x\\ u\end{bmatrix}$

, . . . ,

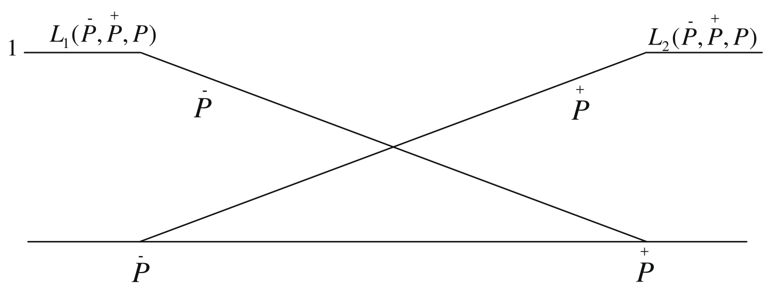

Fig. 1. Membership functions for scheduling variable *P*.

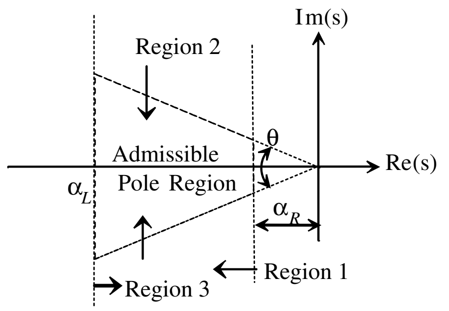

Fig. 2. LMI region: region-1 guarantees an upper bound on the settling time, region-2 guarantees sufficient damping of the system and region-3 prevent controller gains from being excessively large.

Model Rule 8:

$$\text{IF}\quad (P \text{ is about } \overset{+}{P}) AND (Q \text{ is about } \overset{+}{Q}) AND (X_e \text{ is about } \overset{+}{X}_e)$$

$$\text{THEN}\quad \begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} A_8 & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix}$$

The resulting fuzzy system is inferred as the weighted average of the local models and has the form

$$\begin{bmatrix} \dot{x} \\ y \end{bmatrix} = \begin{bmatrix} \left(\sum_{i=1}^{8} \alpha_i A_i \right) & B \\ C & 0 \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} \tag{7}$$

Any value $P \in \left[\bar{P} \quad \overset{+}{P} \right]$ can be expressed as $P = L_1(\bar{P}, \overset{+}{P}, P) \times \bar{P} + L_2(\bar{P}, \overset{+}{P}, P) \times \overset{+}{P}$, where $L_1(\bar{P}, \overset{+}{P}, P)$ and $L_2(\bar{P}, \overset{+}{P}, P)$ are membership functions for the variable P such that $L_1(\bar{P}, \overset{+}{P}, P) + L_2(\bar{P}, \overset{+}{P}, P) = 1$, consequently these membership functions can be calculated as:

$$L_1(\bar{P}, \overset{+}{P}, P) = \frac{\overset{+}{P} - P}{\overset{+}{P} - \bar{P}}, \qquad L_2(\bar{P}, \overset{+}{P}, P) = \frac{P - \bar{P}}{\overset{+}{P} - \bar{P}} \tag{8}$$

The membership functions $L_1(\bar{P}, \overset{+}{P}, P)$ and $L_2(\bar{P}, \overset{+}{P}, P)$ are labeled "$\bar{P}$" and "$\overset{+}{P}$" respectively. Fig. 1 shows the membership functions for the variable P. In a similar manner, membership functions for Q and X_e are defined and labeled M_1, M_2 and N_1, N_2 respectively. The weights are calculated as $h_1 = L_1M_1N_1, h_2 = L_1M_1N_2, h_3 = L_1M_2N_1, \ldots,$ and $h_8 = L_2M_2N_2$.

Remark 1. In the proposed modeling approach, it should be noticed that a single-machine subsystem is approximated by a separate T–S fuzzy model. As result of this approach, a multi-machine power system could be decomposed into a set of T–S fuzzy models that allow for a decentralized design. Interactions between different T–S fuzzy models are guaranteed by a set of scheduling variables (P, Q, X_e) that appear in the premise parts of a model. The sets of different models vary simultaneously and dependently via the network.

3. Representing power system specifications as an LMI Region

In power systems, a damping ratio of at least 10% and a real part not greater than −0.5 guarantees better damping characteristics for low frequency oscillations (Rao & Sen, 2000). These transient response specifications can be satisfied by clustering the closed loop poles in the admissible region shown in Fig. 2. This ensures a minimum decay rate α_R and a minimum damping $\zeta_{\min} = \cos(\theta/$

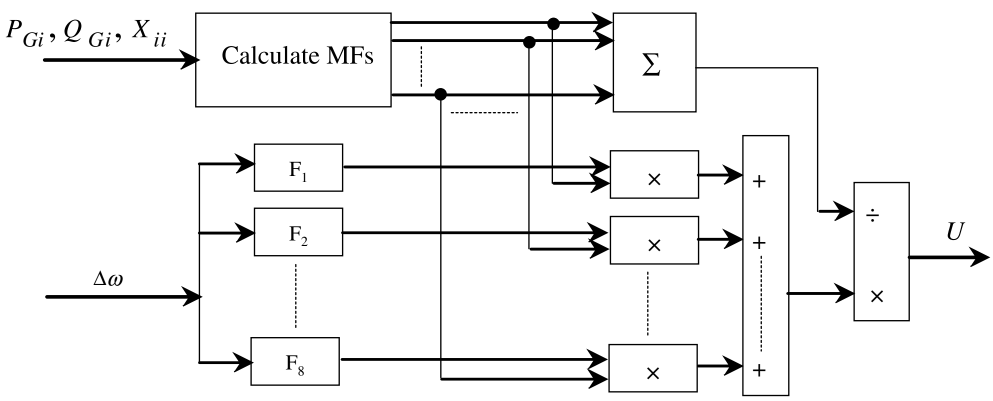

Fig. 3. Schematic diagram for the proposed stabilizer on Gen # i.

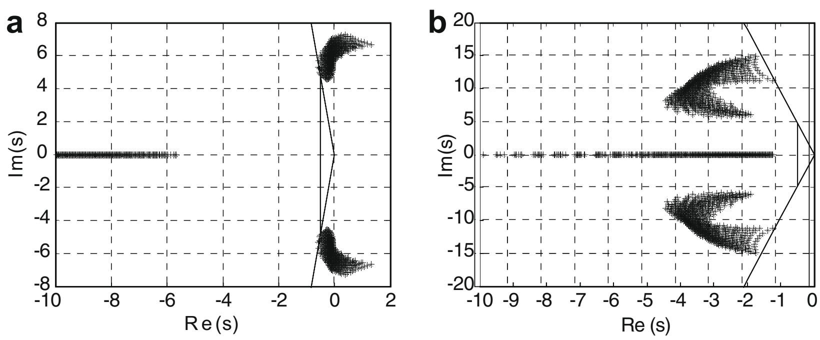

Fig. 4. (a) Dominant open loop poles and (b) dominant closed loop poles with the proposed design.

2). This in turn bounds the maximum overshoot and the settling time of the closed loop system. To avoid very large feedback gains, the real part of the poles should be placed to the right of the α_L line.

The admissible region is expressed as an LMI region defined by three individual LMI regions as shown in Fig. 2. The intersection of the LMI regions results in another LMI region. An LMI region is any subset $D$of the complex plane defined by Chilali, Gahinet, and Apkarian (1999) as follows:

$$D = \{s \in C : \Phi + s\Psi + \bar{s}\,\Psi^{\mathrm{T}} < 0\} \tag{9}$$

where, Φ and Ψ are real matrices and $\Phi = \Phi^T$. The region matrices Φ and Ψ are calculated from the values of α_R, α_L and θ as clarified in Chilali et al. (1999); Chilali and Gahinet (1996). An LMI condition for D-stability of a closed loop system with state matrix A_{cl} is given by the following lemma.

Lemma 1 (Chilali et al., 1999). *The matrix A_{cl} is D-stable if and only if there exists a symmetric, positive definite matrix X such that*

$$\Phi \otimes X + \Psi \otimes (XA_{cl}) + \Psi^{\mathrm{T}} \otimes (XA_{cl})^{\mathrm{T}} < 0 \tag{10}$$

Proof. see Chilali et al. (1999) and Chilali and Gahinet (1996). □

4. Synthesis of a Fuzzy Static Output-Feedback PSS

Typically, a PSS has the speed deviation as a feedback signal. In such case, attention is oriented towards output feedback design

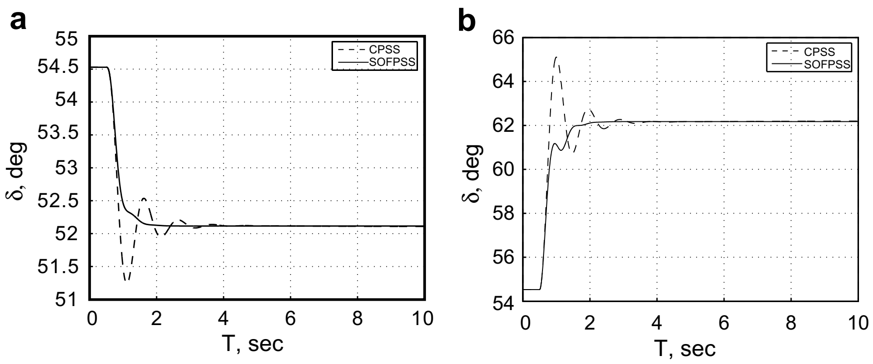

Fig. 5. Rotor angle (deg) at $P = 0.9$, $Q = 0.5$, $X_e = 0.4$ (a) due to 2% step change in reference voltage V_{ref}, (b) due to 20% step change in mechanical torque T_m.

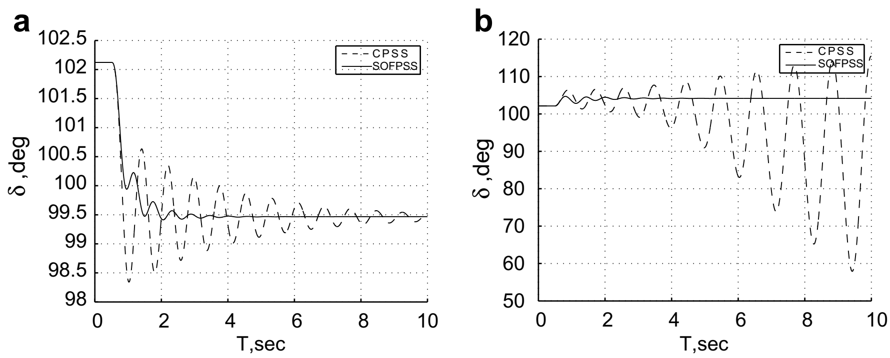

Fig. 6. Rotor angle (deg) at $P = 1.0$, $Q = -0.1$, $X_e = 0.4$ (a) due to 2% step change in reference voltage V_{ref}, (b) due to 10% step change in mechanical torque T_m.

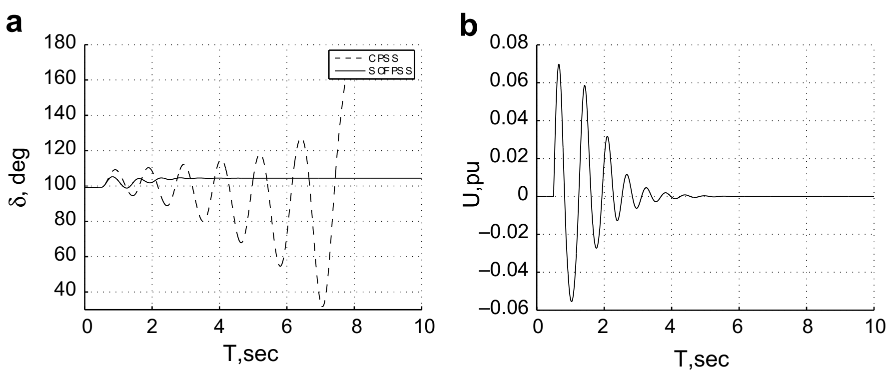

Fig. 7. System response due to 20% step change in mechanical torque T_m at $P = 1.1$, $Q = 0.0$, $X_e = 0.4$ (a) Rotor angle (deg) and (b) control signal (pu) by the proposed design SOFPSS.

methods. This section studies the design of a static output-feedback PSS for power systems described by continuous T–S fuzzy models. Generally the problem of a static output feedback leads to a BMI which is non-convex. Many papers addressed this problem and present iterative LMI techniques to solve this problem, e.g. Cao, Lam, and Suns (1998), He and Wang (2006), Fujimori (2004), Yu (2004) and Haung and Nguang (2006). The authors of Crusius and Trofino (1999) and Chadli et al. (2002) present a solution for the static output feedback via an equality constraint. A fuzzy static output-feedback PSS shares the same fuzzy sets with the fuzzy model as follows:

$$u(t) = \sum_{i=1}^{r} \alpha_i F_i y(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} \alpha_i \alpha_j \{F_i C_j x(t)\} \tag{11}$$

where F_i are the local static output-feedback gains to be determined. By substituting (11) in T–S model (1), we obtain,

$$\dot{x} = \sum_{i=1}^{r} \sum_{j=1}^{r} \sum_{\ell=1}^{r} \alpha_i \alpha_j \alpha_\ell \{A_i + B_i F_j C_\ell\} x(t) \tag{12}$$

For the case of power systems, $B_i = B, C_\ell = C, i, \ell = 1,2, \ldots, r$, then (12) can be rewritten as follows:

$$\dot{x} = \sum_{i=1}^{r} \alpha_i \{A_i + BF_i C\} x(t) \tag{13}$$

The following theorem gives sufficient conditions in LMI form to ensure D-stability of (13).

Theorem 2. *Let $F_i = N_i M^{-1}$, the eigenvalues of (13) lie in the LMI region (9) if there exist matrices $R, M, N_i, i = 1,2, \ldots, r$ such that the following LMIs hold.*

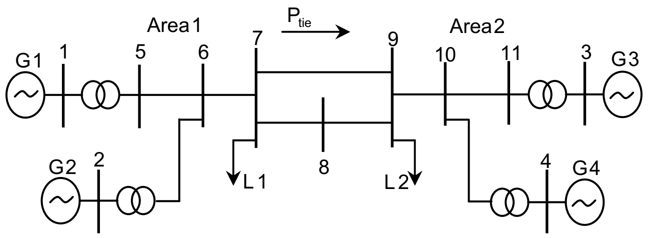

Fig. 8. The four-machine two-area test system.

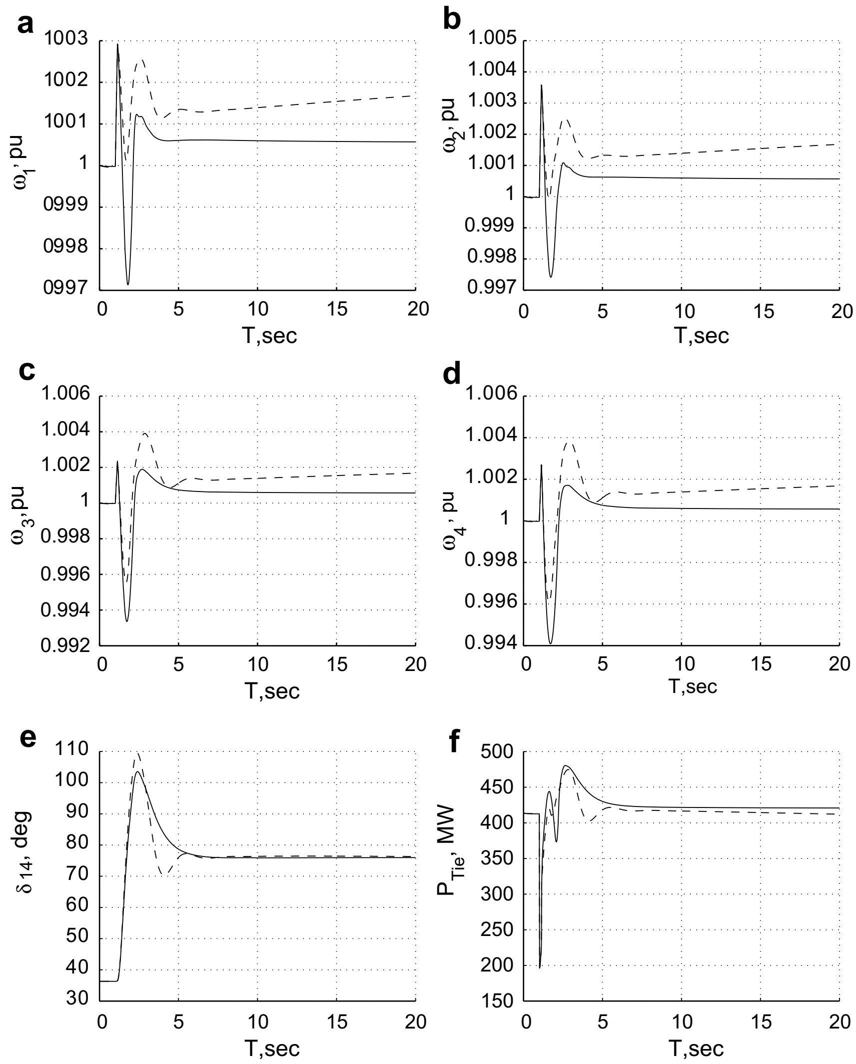

Fig. 9. System response due to three phase short circuit at the middle of one tie line cleared after 0.133 s. (a–d) Rotors speed (pu) for m/cs: 1–4 respectively [– – – CPSS, ——— proposed stabilizer SOFPSS]. (e) Relative rotor angle (deg) between m/c-1 and m/c-4 [– – – CPSS, ——— proposed stabilizer SOFPSS]. (f) tie line power (MW) from area-1 to area-2 [– – – CPSS, ——— proposed stabilizer SOFPSS].

$$R > 0 \tag{14.a}$$

$$\Phi \otimes R + \Psi \otimes (A_i R + BN_i C) + \Psi^{\mathrm{T}} \otimes (A_i R + BN_i C)^{\mathrm{T}} < 0, \quad i = 1, 2, \ldots, r \tag{14.b}$$

$$MC = CR \tag{14.c}$$

Proof. Substituting (13) in (10), we get

$$\Phi \otimes X + \Psi \otimes (X\{A_i + BF_i C\}) + \Psi^{\mathrm{T}} \otimes (X\{A_i + BF_i C\})^{\mathrm{T}} < 0 \tag{15}$$

Performing congruence transformation with $(I \otimes X^{-1})$ on (15) leads to

$$\Phi \otimes X^{-1} + \Psi \otimes (A_i X^{-1} + BF_i CX^{-1}) + \Psi^{\mathrm{T}} \otimes (A_i X^{-1} + BF_i CX^{-1})^{\mathrm{T}} < 0$$

Substituting $X^{-1} = R, CR = MC$ and $F_i M = N_i$ leads to LMIs (13). □

Remark 2. Since the matrix C is full row-rank as the case studied herein, we can deduce from (14.c) that there exists a non-singular matrix $M = CRC^T(CC^T)^{-1}$.

The design steps can be summarized as follows:

(i) Determine the ranges $P \in \left[\bar{P} \quad \overset{+}{P}\right]$, $Q \in \left[\bar{Q} \quad \overset{+}{Q}\right]$ and $X_e \in \left[\bar{X}_e \quad \overset{+}{X}_e\right]$ that encompass all practical operating conditions.

(ii) Define the eight local models of the polytope (6) by calculating $A_1, A_2, \ldots, A_8$, B and C.

(iii) Define the membership functions as given by (8) according to the ranges of P, Q and X_e in (i).

(iv) Generate the T–S fuzzy system (7).

(v) Define α_R, α_L and θ. Then, compute the LMI region matrices Φ and ψ in (9) as clarified in Chilali et al. (1999).

(vi) Solve the optimization problem in (14) to get the static gains of the stabilizer $F_i, i = 1,2,\ldots,8$ using an appropriate LMI solver, e.g. (Gahinet, Nemirovski, Laub, & Chilali, 1995).

(vii) Impalement the control law given by (11) as illustrated in Fig. 3.

5. Design validation and simulation results

The proposed PSS design is validated in this section based on two different nonlinear models. The first model is a single-machine infinite-bus model which is used to illustrate the design steps. The second model is a four-machine two-area system which is used as a benchmark problem in the literature. In applying our design algorithm to the multi-machine system, each machine is considered as a single machine connected to an infinite bus by a tie line. The effect of the rest of the system is reflected on the calculation of the line

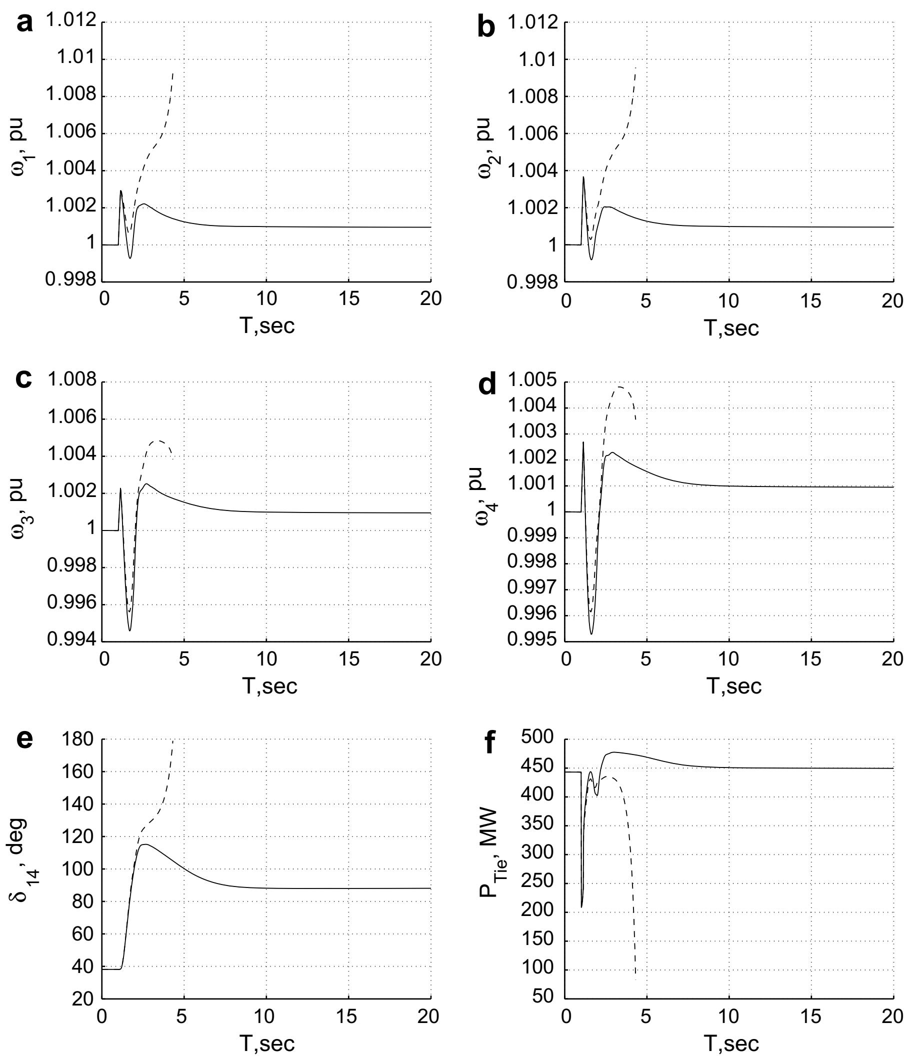

Fig. 10. System response due to three phase short circuit at the middle of one tie line cleared after 0.133 s. (a–d) Rotors speed (pu) for m/cs: 1–4 respectively [– – – CPSS, ——— proposed stabilizer SOFPSS]. (e) Relative rotor angle (deg) between m/c-1 and m/c-4 [– – – CPSS, ——— proposed stabilizer SOFPSS]. (f) Tie line power (MW) from area-1 to area-2 [– – – CPSS, ——— proposed stabilizer SOFPSS].

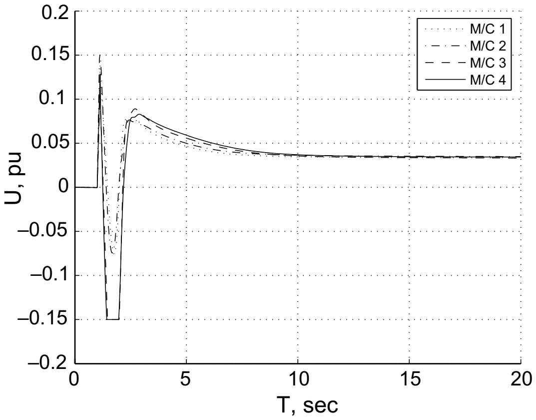

Fig. 11. The control signal (pu) of four machines provided by the proposed design SOFPSS.

reactance and the power delivered to the system. Consequently, a PSS is designed independently for each machine. The implementation details for the proposed stabilizer are shown in Fig. 3.

5.1. The single-machine infinite-bus system

The study in this section will be carried on a single-machine infinite-bus system whose model and data are given in Appendix A.1 P and Q at the generator terminals and X_e are assumed to vary independently over the following ranges provided that all points included have a steady state load flow solution: $P \in [0.4 \;\; 1.0]$, $Q \in [-0.2 \;\; 0.5]$ and $X_e \in [0.2 \;\; 0.4]$. Fig. 4a shows the dominant open loop poles for 1000 plants as P,Q and X_e vary over their specified ranges. It is noted that, most of the plants in this polytope do not have adequate damping and some plants are unstable. The proposed design is carried out for an LMI region bounded by $\alpha_L = -1000$, $\alpha_R = -0.5$ and $\theta = 168°$. The matrices Φ and ψ of the LMI region are computed and listed in Appendix A.2. The optimization problem (14) is solved to calculate the static feedback gains F_i. The resulting values of the stabilizer gains are listed in Appendix A.3. Fig. 4b shows the efficacy of the proposed design in clustering the system roots in the pre-defined LMI region. The time response of three operating conditions is studied and depicted in Fig. 5–7. The CPSS for the same unit is given in Klein et al. (1992) and adopted for comparison. It is obvious that the proposed design outperforms the conventional PSS even at the nominal point. The conventional design fails to maintain stability at full load with leading power factor as shown in Fig. 6 and fails for the case of overload with unity power factor as shown in Fig. 7 as well.

5.2. Decentralized application in a multi-machine power system

The benchmark two-area model shown in Fig. 8 is adopted for simulation studies. The test system consists of two fully symmetrical areas linked together by two 230 KV lines of 220 Km length. It is specifically designed in Kundur (1994) to study low frequency electromechanical oscillations in large interconnected power systems. Each area is equipped with two identical round rotor generators rated 20 KV/900 MVA. The synchronous machines have identical parameters except for the inertias which are $H = 6.5$ s in area 1 and $H = 6.175$ s in area 2. Thermal plants having identical speed regulators are further assumed at all locations, in addition to fast static exciter with a gain of 200. Each generator is represented by a 7th order model. The loads are represented as constant impedances and spilt between the areas. Each generator is equipped with a conventional PSS as designed in Kundur (1994) and Klein et al. (1992) for the same test system. A general procedure to separately design a PSS for each generator includes the following steps:

(i) The load flow study is carried out for different loading conditions that may be encountered during the power system operation to obtain the ranges $P_i \in \left[\bar{P_i} \;\; \overset{+}{P_i}\right]$ and $Q_i \in \left[\bar{Q_i} \;\; \overset{+}{Q_i}\right]$ for different generators, where $i = 1, 2, \ldots, n$ and n is the generator index.
(ii) For different network topologies (normal and contingency conditions are assumed), the bus impedance matrix is calculated, and different self impedances are determined at the generator buses to get $X_i \in \left[\bar{X_i} \;\; \overset{+}{X_i}\right]$, where $i = 1, 2, \ldots, n$ and n is the generator index.
(iii) Once all ranges are determined, the steps described in Section 5 are used to find a T–S fuzzy observer/stabilizer for each generator separately.

The proposed PSS is compared to the conventional stabilizer at two test points. For fair comparison, all simulation results consider saturation limits of ±0.15 pu on the control signals provided either by CPSS or by the proposed stabilizer. Fig. 9 depicts the system response due to a three-phase short circuit at bus-8 when the nominal tie line power is transferred from area-1 to area-2. The fault is cleared after 0.133 s by opening the two breakers at the ends of the faulty line causing one tie-line separation. It is clear that the proposed PSS outperforms CPSS even at the nominal point. If the same fault occurs at larger tie line power, the conventional design fails to maintain system stability, however the proposed PSS gives acceptable damping characteristics as shown in Fig. 10. The control signals of this case provided by the proposed stabilizers to the four machines are depicted in Fig. 11.

6. Conclusion

A design of a power system stabilizer that can cope with a wide range of loading conditions and external disturbances has been the objective of the power industry. This paper has provided a step towards this goal. One of the contributions here has been to show that the nonlinear model of a power system can be systematically represented in the form of a T–S fuzzy system. This has allowed us to use an approximate design model of the power system to develop a stabilizer that copes with different operating conditions and disturbances. Since the fuzzy model is a polytopic system, the proposed design assures stability and performance for all operating points within the polytope.

A static output-feedback fuzzy PSS that guarantees robust pole-placement in an LMI region has been designed. The design conditions have been derived via an LMI approach. Simulation results of a 4-machine 2-area power system have confirmed the superiority of the proposed algorithm in damping the post-fault inter-area oscillations. Compared to a well-tuned conventional PSS, it has been shown that the proposed PSS has a superior capability to cope with larger tie-line power.

Appendix A

A.1. Machine data and model adopted for SMIBB simulation

$$x_d = 1.8, \quad x'_d = 0.3, \quad x_q = 1.7, \quad T'_{do} = 8, \quad M = 13, \quad \omega_o = 377,$$
$$V^{\infty} = 1.0$$
$$K_A = 200, \quad T_A = 0.001, \quad x_e \in [0.2 \quad 0.4]$$
$$\dot{\delta} = \omega_o \omega$$
$$\dot{\omega} = (T_m - E'_q I_q - (x_q - x'_d) I_d I_q)/M,$$

$$\dot{E}'_q = (-E'_q - (x_d - x'_d)I_d + E_{fd})/T'_{do}$$
$$\dot{E}_{fd} = \frac{K_E}{T_E}(V_{ref} - V_T + u_{pss}) - \frac{1}{T_E}E_{fd}$$

where $V_T = \sqrt{V_d^2 + V_q^2} : V_d = -X_e I_q + V^\infty \sin\delta, V_q = X_e I_d + V^\infty \cos\delta$

A.2. The matrices of the LMI Region

$$\Phi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -2000 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad \psi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0.99452 & -0.10453 \\ 0 & 0 & 0.10453 & 0.99452 \end{bmatrix}$$

A.3. The static output-feedback gains

$$F = \begin{bmatrix} 40.895 \\ 38.738 \\ 35.542 \\ 40.319 \\ 47.211 \\ 38.610 \\ 48.256 \\ 45.479 \end{bmatrix}$$

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